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KUNS 1279
July 1994

**Skewness of CMB Anisotropies
in an Inflationary
Isocurvature Baryon Model**

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Abstract

We investigate the cosmic variance of the skewness of the cosmic microwave background (CMB) anisotropies in an inflationary model which leads to the baryon isocurvature scenario for the cosmic structure formation. In this model, the baryon number fluctuations are given by a sinusoidal function of a random Gaussian field. We find that the skewness is very small in comparison with that of the fluctuations which obey Gaussian statistics.

The existence of the nearly isotropic cosmic microwave background (CMB) is a firm evidence of the hot big bang cosmology. At the same time, it is believed that its anisotropies carry valuable information about the early history of our universe. Recently, the COsmic Background Explorer (COBE) satellite team reported the detection of temperature anisotropies in the CMB on scales greater than 10° (Smoot et al. 1992). These fluctuations are particularly important because they directly reflect the gravitational potential fluctuations at $z \sim 1000$. They are hence tightly related to the structure formation of the universe, and their data are powerful clues to know the origin of the cosmic structure.

The angular two-point correlation function is commonly used in a statistical analysis of the temperature fluctuations. However, its knowledge alone cannot distinguish the statistical properties of the fluctuations. To do so, one needs to know at least the next order correlation, i.e, the angular three-point correlation. In the case of the temperature fluctuations which originate from the adiabatic curvature perturbations in the inflationary cosmology, it is shown that its imprint on the angular three-point correlation function is very small (Falk et al. 1993; Gangui et al. 1993), and that the contribution from the cosmic variance will dominate over the signal (Srednicki 1993; Scaramella 1991).

Recently the COBE team reported the results of the data analysis of the three-point correlation function based on the DMR first-year sky map (Hinshaw et al. 1993). In terms of a Monte Carlo analysis with an assumed level of the instrumental noise, they found an evidence of the non-vanishing three-point correlation function in the data which is consistent with the prediction of the standard adiabatic cold dark matter (CDM) model based on the inflationary cosmology. However, it is also true that the low signal-to-noise level in the data prevented them to reproduce the three-point correlation function in a clear form, and hence they could only place an upper limit on the amplitude of it as a firm conclusion. According to them, further several-year observations will produce clear data.

In this situation, it is worthwhile to consider the angular three-point correlation in other possible scenarios of the structure formation besides the standard CDM scenario. Among them, a viable alternative is the Peebles isocurvature baryon (PIB) scenario of the structure formation, in which the universe consists of only baryonic matter and radiation but no non-baryonic dark matter (Peebles 1987), based on the observation $\Omega_0 \sim 0.1$ (Peebles 1986; Tyson 1988).

Furthermore, the second year COBE-DMR data reported recently indicate a power-law index of the primordial density fluctuations which is higher than that of the Harrison-Zeldovich spectrum (Bennett et al. 1994). In addition, several other recent CMB experiments on degree scales have reported the detection of a high amplitude temperature anisotropy (Gundersen et al. 1993; Hancock et al. 1994), which may be regarded as an evidence in favor of the PIB scenario (Hu & Sugiyama 1994), though not all of the results of degree scale experiments seem to be consistent, which may be due to non-Gaussian statistics (Luo 1994).

However, the shortcoming of the PIB scenario, in the theoretical sense, is that it assumes a very *ad hoc* spectrum of the primordial density fluctuations to explain the structure formation and to satisfy the observed isotropy of the CMB on large angular scales, namely isocurvature baryon fluctuations with a steep spectral index.

Now, an interesting microscopic mechanism to produce the seemingly *ad hoc* spectrum

for the PIB scenario has been proposed (Sasaki & Yokoyama 1991; Yokoyama & Suto 1991). In this mechanism, the soft CP violation is induced by a spatially-varying Majoron field, $A(\mathbf{x})$, associated with a heavy Majorana lepton field which decays three quarks or three anti-quarks, and the space-dependent net baryon number,

$$B(\mathbf{x}) = B_* \sin\left(\frac{A(\mathbf{x})}{f}\right) \quad (1)$$

is generated, where f is the mass scale of the Majoron field and B_* is a constant determined by the coupling constants of the model. In particular, Sasaki and Yokoyama (1991) found that a power-law inflation model provides the most appropriate class of power spectra for the PIB scenario. In this case there appears a characteristic scale k_c , which is determined by the model of inflation and the mass scale f , and the power spectrum approaches the white noise as $k/k_c \rightarrow 0$ and almost the scale invariant one as $k/k_c \rightarrow \infty$. Thus it has a nice feature that the amplitude of small scale fluctuations does not become too large on very small scales. Further, they have shown that there exists a natural particle physics model that can provide an appropriate initial condition for the PIB scenario, implemented in power-law inflation with the power-law index around $n_p \simeq 10 \sim 20$. The only difference from the original PIB scenario is that one naturally expect the presence of a cosmological constant that makes the universe spatially flat in this inflationary isocurvature baryon model. Detailed statistical properties of the baryon fluctuations in this model has been studied by Yamamoto et al. (1992). Considering the viability of the PIB scenario, it is important to examine if this model for the PIB scenario has statistical properties which are observationally testable.

In this letter, based on the results of Yamamoto et al. (1992), we consider the skewness of the CMB temperature fluctuations in this scenario. We denote the temperature fluctuation field by $\frac{\Delta T}{T}(\mathbf{x}, \vec{\gamma}_1)$, where \mathbf{x} specifies the position of the observer, the unit vector $\vec{\gamma}_1$ points a given direction from \mathbf{x} . The temperature fluctuation can be evaluated using the isocurvature perturbation theory (Kodama & Sasaki 1986). The baryon number fluctuations correspond to entropy perturbations which give rise to gravitational potential perturbations after the universe becomes matter-dominated. Then the temperature fluctuations directly trace the primordial baryon number fluctuations on large angular scales,

$$\frac{\Delta T}{T}(\mathbf{x}, \vec{\gamma}_1) = cB(\mathbf{x}_1), \quad (2)$$

where c is a proportional constant (Sasaki & Yokoyama 1991), $\mathbf{x}_1 := \mathbf{x} + r_0 \vec{\gamma}_1$, $r_0 = H^{-1} \int_0^1 dy / \sqrt{\Omega_0 y + (1 - \Omega_0)y^4} \sim 2\Omega_0^{-0.4} H_0^{-1}$ ($\Omega_0 \gtrsim 0.1$), and H_0 is the Hubble constant. Thus the statistics of the temperature fluctuation can be understood through that of $B(\mathbf{x})$.

Now we focus on the skewness of the CMB anisotropy,

$$S := \int \frac{d\Omega_{\vec{\gamma}_1}}{4\pi} \left(\frac{\Delta T}{T}(\mathbf{x}, \vec{\gamma}_1) \right)^3. \quad (3)$$

where $d\Omega_{\vec{\gamma}_1}$ denotes the integration over the direction $\vec{\gamma}_1$. As is clear from Eqs.(1) and (3), we have $\langle S \rangle = 0$, where $\langle \rangle$ denotes the ensemble average on the position of observer \mathbf{x} .

Then we consider the cosmic variance of the skewness, which can be written as

$$\langle S^2 \rangle = \int \int \frac{d\Omega_{\vec{\gamma}_1}}{4\pi} \frac{d\Omega_{\vec{\gamma}_2}}{4\pi} \left\langle \left(\frac{\Delta T}{T}(\mathbf{x}, \vec{\gamma}_1) \right)^3 \left(\frac{\Delta T}{T}(\mathbf{x}, \vec{\gamma}_2) \right)^3 \right\rangle. \quad (4)$$

The ensemble average of the temperature fluctuations can be replaced by the correlation of the baryon number fluctuations by using Eq.(2). The random field $B(\mathbf{x})$ follows a peculiar statistic, because it is a sinusoidal function of the Gaussian random field $A(\mathbf{x})$. However, we can evaluate the $2m$ -point correlation function of $B(\mathbf{x})$ by using the following formula that is derived in by Yamamoto et al. (1992),

$$\left\langle \prod_{j=1}^{2m} B(\mathbf{x}_j) \right\rangle \simeq \left[\frac{B_*^2}{4} e^{n_p \beta} \right]^m \sum_{\sigma}' \exp \left[n_p \beta \sum_{j < i} \sigma_j \sigma_i \left(\frac{|\mathbf{x}_i - \mathbf{x}_j|^2}{\eta_f^2} \right)^{1/n_p} \right], \quad (5)$$

where $\beta := H_f^2 / 8\pi^2 f^2$, H_f is the Hubble parameter at the time when the inflation ends, n_p is the power-law index of the power-law inflation, σ_i ($i = 1, 2, \dots, 2m$) takes the value ± 1 , and \sum_{σ}' means that the summation is taken all over the combinations $(\sigma_1, \sigma_2, \dots, \sigma_{2m})$ with $\sum_{j=1}^{2m} \sigma_j = 0$. After straightforward but tedious calculations, we find,

$$\left\langle B(\mathbf{x}_1)^3 B(\mathbf{x}_2)^3 \right\rangle = \frac{1}{4} \left(\frac{B_*^2}{2} \right)^{-6} \left\langle B(\mathbf{x}_1) B(\mathbf{x}_2) \right\rangle^9 + \frac{9}{4} \left(\frac{B_*^2}{2} \right)^2 \left\langle B(\mathbf{x}_1) B(\mathbf{x}_2) \right\rangle. \quad (6)$$

Note the appearance of the 9th power of the two-point correlation in the first term. This term arises because there are 3×3 independent combinations between \mathbf{x}_i ($i = 1, 2, 3$) and \mathbf{x}_j ($j = 4, 5, 6$) before we take the coincidence limits. The two-point correlation of $B(\mathbf{x})$ is given by (Sasaki & Yokoyama 1991)

$$\left\langle B(\mathbf{x}_1) B(\mathbf{x}_2) \right\rangle = \int \frac{d^3 \mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}(\mathbf{x}_1 - \mathbf{x}_2)} P_B(k), \quad (7)$$

where

$$P_B(k) = 2\pi B_*^2 e^{n_p \beta} \frac{1}{k^3} \int_0^\infty ds s \sin s \exp \left[- \left(\frac{s}{k/k_c} \right)^{2/n_p} \right]. \quad (8)$$

The ensemble average of the angular two-point correlation function is given by $C(\alpha) := \left\langle \frac{\Delta T}{T}(\mathbf{x}, \vec{\gamma}_1) \frac{\Delta T}{T}(\mathbf{x}, \vec{\gamma}_2) \right\rangle = c^2 \left\langle B(\mathbf{x}_1) B(\mathbf{x}_2) \right\rangle$, where $\cos \alpha = \vec{\gamma}_1 \cdot \vec{\gamma}_2$. This is written in terms of the multipole moments as

$$C(\alpha) = \frac{1}{4\pi} \sum_l (2l+1) \left(c^2 \frac{2}{\pi} \int_0^\infty dk k^2 j_l(kr_0)^2 P_B(k) \right) W_l^2 P_l(\cos \alpha). \quad (9)$$

Here the window function $W_l = \exp(-l(l+1)/2\sigma^2)$, $\sigma = 17.8$, is inserted in order to compare it with the result of COBE (Smoot et al. 1992).

Then, from Eqs.(4),(6) and (9), we obtain

$$\langle S^2 \rangle = \frac{1}{8} \left(\frac{B_*^2}{2} c^2 \right)^{-6} \int_{-1}^1 d(\cos \alpha) C(\alpha)^9, \quad (10)$$

where we have used the fact that the monopole term (and also the dipole term) in the temperature fluctuations should be subtracted from the data. The variance of the skewness in the Gaussian statistics takes a similar form (Srednicki 1993), but differs from the present case in the powers of $C(\alpha)$.

For comparison with other papers we normalize the variance of skewness by $C(\alpha = 0)^3$, and consider the normalized root mean square skewness, *i.e.*, $\hat{S}_{r.m.s.} := \langle S^2 \rangle^{1/2} / C(0)^{3/2}$. We can show that this can be written as

$$\hat{S}_{r.m.s.} = \left(\frac{\sqrt{2}}{\pi} e^{n_p \beta} \right)^3 \mathcal{A}, \quad (11)$$

where

$$\mathcal{A} := \left[\frac{\int_{-1}^1 d(\cos \alpha) \left\{ \sum_l (2l+1) d_l W_l^2 P_l(\cos \alpha) \right\}^9}{\left\{ \sum_l (2l+1) d_l W_l^2 \right\}^3} \right]^{1/2}, \quad (12)$$

$$d_l := \int_0^\infty \frac{dk}{k} j_l(kr_0)^2 \int_0^\infty ds s \sin s \exp \left[- \left(\frac{s}{k/k_c} \right)^{2/n_p} \right]. \quad (13)$$

For the present scenario to be successful, we assume $n_p \beta < 1$ (Sasaki & Yokoyama 1991). Hence, the coefficient of \mathcal{A} in the Eq.(11) is of the order of unity. Therefore we focus on the numerical factor \mathcal{A} . If the two parameters, *i.e.*, n_p (the exponent of the power-law inflation) and $x_c := r_0 k_c$ (the ratio of the present horizon size r_0 to the characteristic scale $1/k_c$) are specified, we can calculate it numerically. The Table 1 shows the results of numerical integration of \mathcal{A} for various values of x_c in the case $n_p = 10$ and the Table 2 does in the case $n_p = 20$.

Once n_p and x_c are fixed, we can also obtain the angular two-point correlation function $C(\alpha)$ from Eq.(9). We numerically fitted this angular two-point correlation function $C(\alpha)$ normalized by $C(0)$ to that due to the density fluctuation with the power-law spectrum $\langle (\Delta\rho/\rho)_k^2 \rangle \propto k^{n_{\text{eff}}}$, and calculated the best fitted value of n_{eff} . Thus n_{eff} is the effective spectral index of the fluctuations on the COBE scale; $n_{\text{eff}} \gtrsim 2$ for $x_c \gg 1$ while $n_{\text{eff}} \sim 1$ for $x_c \sim 1$.

Let us roughly evaluate an upper limit on \mathcal{A} . In the case $x_c \sim 1$, for which the spectrum is almost scale invariant, we have $d_l \simeq \pi/2l(l+1)n_p$. Then we find $\mathcal{A} \lesssim O(n_p^{-3})$. However, this case is not appropriate for the PIB scenario. When $x_c \gg 1$, which is the case of our interest, the value of \mathcal{A} is very small as shown in the Tables 1 and 2. Thus the cosmic variance of the skewness turns out to be very small in comparison with the one obtained for the Gaussian statistics with the Harrison-Zeldovich spectrum, $\hat{S}_{r.m.s.} \simeq 0.18$ (Srednicki 1993). This suggests that the cosmic variance of the angular three-point correlation function is also very small. We suspect that the higher order correlation is always suppressed in this model.

If we accept the data analysis of the COBE team (Hinshaw et al. 1993), *i.e.*, the detection of the nonvanishing three-point correlation as they claim, it is difficult to explain it within the present model of the PIB scenario. However, either the actual amplitude of the skewness or the form of the three-point correlation function has not been obtained due

to the high noise level. According to them, the noise level of the three-point correlation function diminishes in proportion to $(time)^{-3/2}$ as the data accumulate. It is necessary for us to wait for a few more years before a definite conclusion may be drawn.

Acknowledgement

K.Y. is grateful to Professor H. Sato for his encouragement. This work was supported in part by Monbusho Grant-in-Aid for Scientific Research No.05640342, and the Sumitomo Foundation.

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Table 1

Skewness and effective spectral index for $n_p = 10$.

x_c	\mathcal{A}	n_{eff}
10^6	3.2×10^{-13}	2.6
10^5	3.0×10^{-9}	2.1
10^4	1.4×10^{-6}	1.7
10^3	5.5×10^{-5}	1.4
10^2	3.6×10^{-4}	1.17
10	7.3×10^{-4}	1.04

Table 2

Skewness and effective spectral index for $n_p = 20$.

x_c	\mathcal{A}	n_{eff}
10^{11}	2.8×10^{-14}	1.9
10^9	6.5×10^{-10}	1.6
10^7	3.7×10^{-7}	1.33
10^5	1.0×10^{-5}	1.17
10^3	5.6×10^{-5}	1.07
10	9.6×10^{-5}	1.01